Optimal 3-Stage Schedule for specially Structured Flow Shop Problem to Minimize the Rental Cost including Transportation and Job Weightage

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Abstract-This article describe the development of a new heuristic algorithm which guarantees an optimal solution for specially structured flow shop problem with n-job, 3-machine , to minimize the rental cost under specified rental policy in which processing times , weightage of jobs ,transportation times are being consider. Further the processing times are not merely random but bear a well defined relationship to one another. Many heuristic approaches have already been discussed in literature to minimize the makespan. But it is not necessary that minimization of makespan always lead to minimize the rental cost of machines. Objective of this work is to minimize the rental cost of machines under a specified rental policy. A numerical illustration is followed to support the algorithm.

Keywords: Flow Shop Scheduling, Rental Cost, Weightage of job, Utilization Time, Transportation Time.

1. Introduction

Scheduling can be defined as the allocation of resources over a period of time to perform a collection of tasks. The goal is to specify a schedule that specify when and on which machine each job is to be executed. A variety of approaches have been developed to solve the problem of scheduling. Majority of research in scheduling assumes transportation time (loading time, moving time and unloading time) from one machine to another as negligible or included in processing time. But in some real life situations transportation time has great impact on the performance measure, separate consideration is needed. All the scheduling models beginning from Johnson's work in 1954 upto 1980, there is no reference of job weightage in the literature. The scheduling problem with weights arises.

The scheduling problem with weights arises when inventory costs for jobs are involved. The weights of a job show its relative priority over some other jobs in a scheduling model. The first research concerned to the flow shop scheduling problem was proposed by Johnson [1]. Johnson described an exact algorithm to minimize make span for the n-jobs 2- stage flow shop scheduling problem. Smith [3] considered minimization of mean flow time and maximum tardiness. Yoshida and Hitomi [8] further considered the problem with set up times. Gupta, J.N.D. [5] gave an algorithm to find the optimal schedule for specially structured flow shop scheduling. Gupta [19] studied specially structured two stage flow shop problem to minimize the rental cost of the machines under pre-defined rental policy

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in which the probabilities have been associated with processing time. Maggu & Das [7] consider a two machine flow shop problem with transportation time. The work was developed by Bhambani [16], Gupta Deepak et. al [13], Maggu & Dass [7], Chader Sekharan [10], Singh T.P.[11] considering various parameters. Gupta [19] studied 3-Stage specially structured flow shop scheduling to minimize the rental cost including job weightage.The present paper is an attempt to extend the study made by Gupta, [18, 19] by introducing transportation time of three machines. Thus the problem discussed in this paper becomes wider and more practical in process/ production industry.

2. Practical Situation

Manufacturing industries are the backbone in the economic structure of a nation, as they contribute to increasing G.D.P. / G.N.P. and providing employment. Productivity can be maximized, if the available resources are utilized in an optimized manner. Optimized utilization of resources can only be possible if there is a proper scheduling system making scheduling a highly important aspect of a manufacturing system. The practical situation may be taken in a paper mill, sugar factory and oil refinery etc. where various qualities of paper, sugar and oil are produced with relative importance i.e. weight in jobs, hence weightage of jobs is significant. Transportation becomes significant when the machines on which jobs are to be processed are placed at different places. Due to unavailability of funds in real life in his starting career one has to be taken the machines on rent. To start a fitness centre many machines like tread mill, laser hair removal equipment, ski cabin, cardiovascular, stretches, free weights, elliptical, cycles, rowers, plate loaded & benches, multi station, one does not want to invest huge money by buying all the machines, instead he prefer to take the machines on rent. By renting one can stay financially afloat more easily and still manage to procure the best high technology for customers.

3. Notations

S : Sequence of jobs 1, 2, 3… n
S_k : Sequence obtained by applying Johnson's procedure, k = 1, 2, 3, ------

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M_j : Machine j, i= 1, 2, 3
a_{ij} : Processing time of i^{th} job on machine M_j.
$t_{ij}(S_k)$: Completion time of i^{th} job of sequence S_k on machine M_j
$I_{ij}(S_k)$: Idle time of machine M_j for job i in the sequence S_k.
$T_{i,j\rightarrow k}$:Transportation time of i^{th} job from j^{th} machine to k^{th} machine.
$U_j(S_k)$: Utilization time for which machine M_j is required
$R(S_k)$: Total rental cost for the sequence S_k of all machine.
C_j : Rent cost per unit time of machine M_j.
w_i : Weight of i^{th} jobs.

4. Definition

Completion time of i^{th} job on machine M_j is denoted by t_{ij} and is defined as:

$t_{ij} = \max(t_{i-1,j}, t_{i,j-1}) + T_{i,j-1\rightarrow j} + a_{ij}$ for $j \geq 2$.

5. Rental Policy

The machines will be taken on rent as and when they are required as and when they are no longer required. i.e. the first machine will be taken on rent in the starting of the processing the jobs and 2^{nd} machine will be taken on rent at time when 1^{st} job is completed on the 1^{st} machine and transported to 2^{nd} machine and 3^{rd} machine will be taken on rent when 1^{st} job is completed on 2^{nd} machine and transported to 3^{rd} machine.

6. Problem Formulation

Let some jobs i(1,2,……..,n) are to be processed on three machines M_j (j= 1,2,3) under the specified rental policy (P). Let a_{ij} be the processing time of i^{th} job on j^{th} machine and $T_{ij\rightarrow k}$ be the transportation time of i^{th} job from j^{th} machine to k^{th} machine. w_i be the weight of i^{th} job. Our aim is to find the sequence $\{S_k\}$ of the jobs which minimize the rental cost of all the machines. The mathematical model of the problem in matrix form can be stated as:

Jobs	Machine M_1	$T_{i1\rightarrow 2}$	Machine M_2	$T_{i2\rightarrow 3}$	Machine M_3	Weight
I	a_{i1}	t_i	a_{i2}	g_i	a_{i3}	w_i
1	a_{11}	t_1	A_{12}	g_1	a_{13}	w_1
2	a_{21}	t_2	A_{22}	g_2	a_{23}	w_2
3	a_{31}	t_3	A_{32}	g_3	a_{32}	w_3
.	.	.	.	.	.	.
.	.	.	.	.	.	.
n	a_{n1}	t_n	a_{n2}	g_n	a_{n3}	w_n

Table – 1

Mathematically, the problem is stated as:
Minimize

$$R(S_k) = \sum_{i=1}^{n} A_{i1} \times C_1 + U_2(S_k) \times C_2 + U_3(S_k) \times C_3$$

Subject to constraint: Rental Policy (P)
Our objective is to minimize rental cost of machines while minimizing the utilization time.

7. Assumptions

1. Jobs are independent to each other. Let n jobs be processed through three machines M_1, M_2, M_3 in order $M_1M_2M_3$.
2. Machine break down is not considered.
3. Pre-emption is not allowed.
4. Either $A_{i1} + t_i \geq A_{j2} + t_i$ for all i, j
   Or $Ai_2 + g_i \leq A_{j3} + g_i$ for all i, j Or both.

8. Algorithm

The algorithm to minimize the rental cost is as follows:

Step 1: Convert the problem into two machines problem. Let G and H be fictious machines having Gi and Hi as their processing times as:

$G_i = A_{i1} + t_i + A_{i2} + g_i$
$H_i = t_i + A_{i2} + g_i + A_{i3}$

Step 2: If min $(G_i, H_i) = G_i$

Then $G_i' = \frac{G_i + w_i}{w_i}$, $H_i' = \frac{H_i}{w_i}$

If min $(G_i, H_i) = H_i$

Then $G_i' = \frac{G_i}{W_i}$, $H_i' = \frac{H_i + w_i}{w_i}$

Step 3: Define a new reduced problem with the processing time G_i and H_i.

Step 4: Apply Johnson's (1954) technique and obtain the optimal schedule of given jobs. Let the sequence be S_1.

Step 5: Obtain other sequences by putting 2^{nd}, 3^{rd},…..n^{th} jobs of sequence S_1 in the 1^{st} position and all other jobs of S_1 in same order. Let these sequences be S_2 ,S_3…S_{n-1}.

Step 6: Compute $\sum A_{i1}$, $U_2(S_k)$, $U_3(S_k)$ and

$$R(S_k) = \sum_{i=1}^{n} A_{i1} \times C_1 + U_2(S_k) \times C_2 + U_3(S_k) \times C_3$$

For all possible sequences S_k (k = 1, 2, …, n)

Step 7: Find min $R(S_k)$; k= 1,2,….,n. let it be minimum for the sequence S_p, then sequence S_p will be the optimal sequence with rental cost $R(S_p)$.

9. Numerical Illustration

Consider 5 jobs and 3 machines flow shop problem in which processing times including transportation time and job weightage are given in the Table- 2. The rental cost per units time for machines M_1, M_2 and M_3 are 10 units, 5 units and 2 units respectively. Our objective is to obtain optimal a sequence of jobs with minimum possible rental cost of the machines.

Solution: Check conditions:
Here max $(A_{i2} + g_i) \leq$ min $(A_{i3} + g_i)$ satisfies. Therefore, as per Step 1 the processing time for two fictitious machines G and H as shown in table 3

Table – 2
The processing time for two fictitious machines G and H:

Jobs	G	H	Weight
i	G_i	H_i	w_i
1	42	81	5
2	44	60	2
3	45	87	3
4	64	104	2
5	64	82	4

Table : 3

As per Step 2 and Step 3: we get new reduced problem with weighted flow times G_i' & H_i' as follows

Jobs	Machine M_1	$T_{i1\to2}$	Machine M_2	$T_{i2\to3}$	Machine M_3	Weight
I	A_{i2}	t_i	A_{i1}	g_i	A_{i3}	w_i
1	10	5	25	2	64	5
2	8	3	32	1	48	2
3	12	2	28	3	70	3
4	14	4	40	6	80	2
5	21	2	36	5	54	4

Jobs i	G_i'	H_i'
1	9.4	16.20
2	23	30
3	16	29
4	33	52
5	17	20.5

Table - 4

As per Step 4: Obtaining the sequence with minimum makespan is S_1 : 1 – 3 – 5 – 2 – 4.

As per Step 5: Other feasible sequences which may corresponds to minimum rental cost are :

S_2 = 3 – 1 – 5 – 2 – 4 , S_3 = 5 – 1 – 3 – 2 – 4
S_4 = 2 – 1 – 3 – 5 – 4 , S_5 = 4 – 1 – 3 – 5 – 2

From in – out tables for these sequences, we have:
For S_1: $CT(S_1) = 373$; $U_2(S_1) = 149$; $U_3(S_1) = 331$; $R(S_1) = 3017$
For S_2: $CT(S_2) = 375$; $U_2(S_2) = 149$; $U_3(S_2) = 330$; $R(S_2) = 3015$
For S_3: $CT(S_3) = 392$; $U_2(S_3) = 141$; $U_3(S_3) = 328$; $R(S_3) = 2971$
For S_4: $CT(S_4) = 376$; $U_2(S_4) = 144$; $U_3(S_4) = 332$; $R(S_4) = 2994$
For S_5: $CT(S_5) = 391$; $U_2(S_5) = 128$; $U_3(S_5) = 327$; $R(S_5) = 2894$

Therefore min $R\{S_K\} = R(S_5) = 2894$ units.
Therefore minimum rental cost is 2894 units and is for the sequence S_5.
Hence the sequence S_5 = 4 – 1 – 3 – 5 – 2 is optimal sequence with minimum rental cost. But total elapsed time / completion time for S_5 is not minimum.

10. Conclusion

The algorithm proposed in this paper to minimize the rental cost of machines given an optimal sequence having minimum rental cost of machines irrespective of total elapsed time. The algorithm proposed by Johnson's (1954) to find an optimal sequence to minimize the makespan/ total elapsed time is not always corresponds to minimum rental cost of machines under a specified rental policy. Further the work can be extended by introducing parameters like job block, break down interval, set-up etc.

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